Sample questions are given below

1. Calculate $\int_0^\pi (x \sin x)^2 dx$.

2. A certain medical condition affects one in a thousand of the population. A screening test is developed that is 95% accurate (in the sense that the probability of testing positive given you have the condition is 0.95, and you may assume that the test only has two outcomes, positive and negative). Discuss the probability that you have the condition if the screening procedure says you do.

3. Investigate the integral

$$\int_0^3 \frac{1}{(1-x)^2} dx$$

4. In a tennis tournament there are $2n$ participants. In the first round of the tournament, each player plays exactly once, so there are n games. Show that pairings for the first round can be arranged in exactly $(2n-1)!/2^{(n-1)}(n-1)!$ ways.

5. If a ball is thrown with speed V at an angle α to the horizontal, show that in polar coordinates (r, θ) the trajectory is given by

$$r = \frac{2V^2 \cos\alpha \sin(\alpha - \theta)}{g\cos^2\theta}$$

By differentiating the above expression for r, verify that the maximum range of a projectile occurs when it is released at an angle of $45°$.

6. One end of a rod of uniform density is attached to the ceiling in such a way that the rod can swing about freely with no resistance. The other end of the rod is held still so that it touches the ceiling as well. Then the second end is released. If the length of the rod is l metres and gravitational acceleration is g ms^{-2}, how fast is the unattached end of the rod moving when the rod is first vertical?

7. The resistance between points A and B is given by the arrangement of individual resistors R_1, R_2, R_3 given in Figure 1a. If we now replace this with the new arrangement of Figure 1b, find the value of R_p such that the total resistance R_{AB} is the same in both cases.

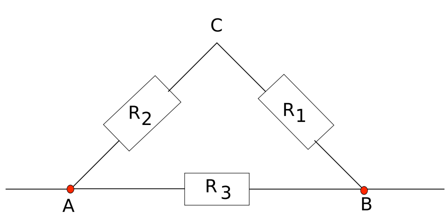

Figure 1a

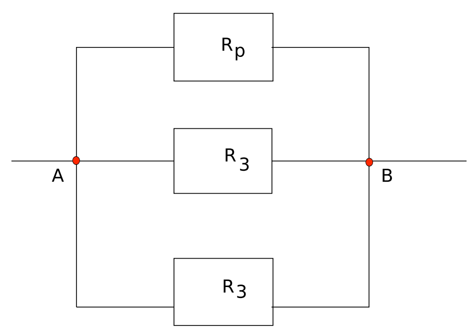

Figure 1b

8. In the circuit shown in Figure 2 calculate the value (shown as X in figure) of resistor R_{BE} if the power taken by the 6V battery is 6.6W. All resistances shown are in Ohms.

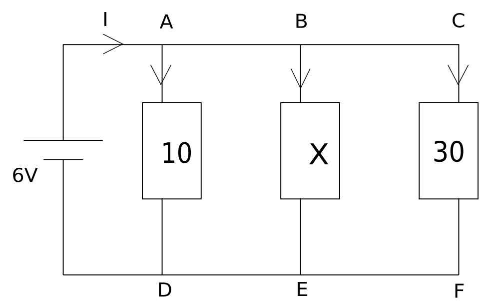

Figure 2

9. Let L_1 and L_2 be two lines in the plane, with equations $y = m_1x + c_1$ and $y = m_2x + c_2$ respectively. Suppose that they intersect at an acute angle θ. Show that

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

10. A particle of mass m is released from rest at a height h above the ground. Neglecting air resistance and assuming gravitational acceleration g is constant, show by dimensional analysis that the time taken to hit the ground is given by $T = C\sqrt{h/g}$ where C is a dimensionless constant.

Now, if air resistance produces a force equal to $-k$ times the velocity of the particle, so that T now depends on m, g, h and k, again use dimensional arguments to show that

$$T = \frac{m}{k}\tau(\lambda)$$

where $\lambda = k^2h/m^2g$ and $\tau(\lambda)$ is a dimensionless unknown function of the dimensionless quantity λ.